Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					



General Certificate of Education Advanced Level Examination June 2010

Mathematics

MFP3

Unit Further Pure 3

Friday 11 June 2010 9.00 am to 10.30 am

For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed

• 1 hour 30 minutes

Instructions

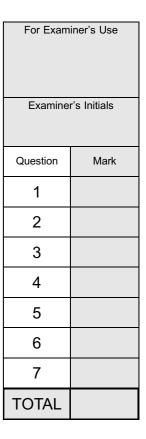
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.



Answer all questions in the spaces provided.

1 The function y(x) satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where

$$f(x, y) = x + 3 + \sin y$$

and

$$y(1) = 1$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with h = 0.1, to obtain an approximation to y(1.1), giving your answer to four decimal places. (3 marks)

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$$

with your answer to part (a), to obtain an approximation to y(1.2), giving your answer to three decimal places. (3 marks)

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2 (a)	Find the value of the constant k for which $k \sin 2x$ is a particular integral of the	e
	differential equation	

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y = \sin 2x \tag{3 marks}$$

(b) Hence find the general solution of this differential equation. (4 marks)

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6

3 (a	Explain why $\int_{1}^{\infty} 4xe^{-4x} dx$ is an improper integral.	(1 mark)
(b	Find $\int 4x e^{-4x} dx.$	(3 marks)
(c	Hence evaluate $\int_{1}^{\infty} 4xe^{-4x} dx$, showing the limiting process used.	(3 marks)
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4 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + \frac{3}{x}y = (x^4 + 3)^{\frac{3}{2}}$$

given that
$$y = \frac{1}{5}$$
 when $x = 1$.

(9 marks)

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- Write down the expansion of $\cos 4x$ in ascending powers of x up to and including the term in x^4 . Give your answer in its simplest form. (2 marks)
 - **(b) (i)** Given that $y = \ln(2 e^x)$, find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

(You may leave your expression for $\frac{d^3y}{dx^3}$ unsimplified.) (6 marks)

(ii) Hence, by using Maclaurin's theorem, show that the first three non-zero terms in the expansion, in ascending powers of x, of $\ln(2 - e^x)$ are

$$-x - x^2 - x^3 \tag{2 marks}$$

(c) Find

$$\lim_{x \to 0} \left[\frac{x \ln(2 - e^x)}{1 - \cos 4x} \right] \tag{3 marks}$$

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6 The polar equation of a curve C_1 is

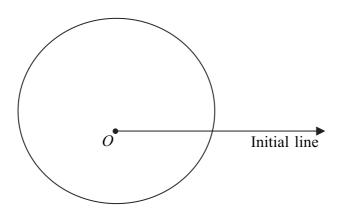
$$r = 2(\cos\theta - \sin\theta), \quad 0 \leqslant \theta \leqslant 2\pi$$

(a) (i) Find the cartesian equation of C_1 .

(4 marks)

- (ii) Deduce that C_1 is a circle and find its radius and the cartesian coordinates of its centre. (3 marks)
- (b) The diagram shows the curve C_2 with polar equation

$$r = 4 + \sin \theta$$
, $0 \le \theta \le 2\pi$



- (i) Find the area of the region that is bounded by C_2 . (6 marks)
- (ii) Prove that the curves C_1 and C_2 do not intersect. (4 marks)
- (iii) Find the area of the region that is outside C_1 but inside C_2 . (2 marks)

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7 (a) Given that $x = t^{\frac{1}{2}}$, x > 0, t > 0 and y is a function of x, show that:

(i)
$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t};$$
 (2 marks)

(ii)
$$\frac{d^2y}{dx^2} = 4t\frac{d^2y}{dt^2} + 2\frac{dy}{dt}$$
. (3 marks)

(b) Hence show that the substitution $x = t^{\frac{1}{2}}$ transforms the differential equation

$$x\frac{d^2y}{dx^2} - (8x^2 + 1)\frac{dy}{dx} + 12x^3y = 12x^5$$

into

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - 4\frac{\mathrm{d}y}{\mathrm{d}t} + 3y = 3t \tag{2 marks}$$

(c) Hence find the general solution of the differential equation

$$x\frac{d^2y}{dx^2} - (8x^2 + 1)\frac{dy}{dx} + 12x^3y = 12x^5$$

giving your answer in the form y = f(x). (7 marks)

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